Bond Pricing Introduction

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Contents

Chapter 1

Part I: Fundamental Bonds Concepts

Bond pricing is the foundation of fixed income markets, requiring mastery of *time value of money, yield calculations, risk metrics*, and *market conventions.*

Why did I decide to actually make a document? Because I want to end up in a trading floor, and this is the way I study. The bond market is significantly larger than equity markets globally, with over $130 trillion in outstanding debt securities (and it is arguably the first stepping stone for any pricing course, namely the first lessons in my Computational Finance course at the University of Padua, hence the mathematics is fairly simple). Bonds trade primarily over-the-counter (OTC) rather than on exchanges, requiring a specialized level of knowledge of market microstructure, liquidity dynamics, and dealer-client relationship where both parties are symmetrically informed.

1.1 What is a Bond?

A **bond**[1] is a debt security representing a loan made by an investor to a borrower, typically a corporation or government. When you purchase a bond, you are essentially lending money to the issure in exchange for periodic interest payments (*coupons*) and the return of the bond's face value (*principal*) at a predefined date (*maturity*).

[1]From the English verb *"to bind"* because historically a bond represented a **binding** agreement between two parties. In medieval and contemporary English law, it was a written obligation acknowledging debt, sealed and witnessed.

Bonds have three essential characteristics:

- **Face Value (Par Value)**: The amount the bond will pay at maturity, in the order of hundreds or thousands for corporate bonds, though institutional trades often involve millions in notional value.
- **Coupon Rate**: The annual interest rate paid by the bond issuer, expressed as a percentage of face value. A $1000 bond with a 5% coupon pays $50 annually.
- **Maturity Date**: The date when the principal must be repaid to the holder of the bond (from here on out, referred as *investor* or *lender*).

1.2 Bond Types and Credit Quality

Bonds are categorized by issuer type and credit quality.
Government Bonds are bonds issued by national governments (U.S. Treasuries, German Bunds, Italian BTPs, Japanese JGBs…). These are considered virtually risk-free in developed markets and serve as the benchmark for pricing other fixed income securities.
Corporate Bonds are bonds issued by companies to finance operations, acquisitions, or capital expenditures. These can be further subdivided into:

- **Investment Grade (IG)**: Rated ≥*BBB-/Baa3* by credit rating agencies, indicating lower default risk.
- **High Yield (HY)**: Rated <*BBB-*, also called *junk bonds* offer higher yields to compensate the greater default risk.

Municipal Bonds: issued by state and local governments in the U.S., often with tax-exempt interest income for investors. A greater distinction exist between **GSEs**[2]-issued bonds, for example those issued by the *Federal National Mortgage Association* (a.k.a. *Fannie Mae* or FNMA) and the *Federal Home Loan Mortgage Corporation* (a.k.a *Freddie Mac* or FHLMC), where government backing can be implicit or explicit.
Mortgage-Backed Securities (MBS) are bonds that are essentially backed by pools of mortgages, subject to prepayment risk when homeowners refinance.

MBSs were the central mechanism behind the 2008 financial crisis.

1.3 How Bonds Work: Cash Flows

A bond's entire structure revolves around its **cash flows**, which are known in advance and contractually guaranteed by the issuer (unless, of course, the issuer defaults). These cash flows consist of two components: periodic *coupon payments* and the repayment of

[2] government-sponsored entities

principal at maturity. The pattern and timing of these payments are the foundation of bond analysis and pricing.

Most conventional bonds pay coupons *semi-annually*. If the coupon rate is 5% on a bond with a face value of $1000, then:

- Every 6 months the investor receives $\frac{(\$1000\times 5\%)}{2} = \$25$
- At maturity (after 10 semi-annual periods), the lender receives $\$25 + \$1000 = \$1025$.

Cash Flow Schedule Example

Let us illustrate this structure. For a 5-year semiannual 5% bond, the stream of cash flows is perfectly predictable:

Period (6 months)	**Cash Flow ($)**
1	25
2	25
3	25
4	25
5	25
6	25
7	25
8	25
9	25
10	1025

These payments occur at equal intervals, and this allows investors to forecast and value them with mathematical precision. The regularity of these cash flows is what makes bonds "fixed income" instruments, the payments are fixed and scheduled, unlike equity dividends that depend on company performance.

Coupon Frequency and Conventions

The **frequency of coupon payments** varies by market convention:

- **Annual**: Common in Eurozone sovereign and corporate markets.
- **Semi-annual**: Standard for U.S. Treasury and corporate bonds.
- **Quarterly or Monthly**: Found in floating-rate notes (FRNs) and certain structured products such as mortgage-backed securities.

1.3.1 Day Count and Accrued Interest

As explained in later sections, between coupon dates, bond prices must account for **accrued interest**, which basically is the portion of the next coupon payment that has already been earned but not yet paid. If a bond is sold midway between coupon dates, the buyer compensates the seller for this earned interest. Accrued interest conventions can vary across markets (e.g. *Actual/Actual*, *30/360*), but the logic remains identical: interest accrues linearly between payments.

1.3.2 Zero-Coupon and Other Variations

Zero-Coupon bonds represent the simplest case of bond cash flows, with no periodic payments, only a single lump sum at maturity equal to the face value. These bonds are typically issued at a discount, making the difference between purchase and maturity price the investor's return.
Other variations include:

- **Floating-rate notes (FRNs)**: Coupons reset periodically based on a reference rate (e.g. LIBOR, EURIBOR, or SOFR) plus a fixed spread.
- **Inflation-linked bonds**: Coupon and/or principal payments adjust according to inflation indices such as the CPI.

Regardless of these variations, every bond can be reduced to a stream of deterministic (or conditionally deterministic) cash flows.

Chapter 2

Part II: Time Value of Money and Bond Valuation

2.1 The Foundation: Present Value

The core principle underlying bond pricing is the **time value of money**: we know this commonly by the saying "*a dollar today is worth more than a dollar in the future*". Obviously because the dollar of today can be invested to earn returns.
To value a bond, we must calculate the Present Value (PV) of all future cash flows. The **present value formula** for a single future cash flow is:

$$PV = \frac{CF}{(1+r)^t} \tag{2.1}$$

where we identify the classic time value discount based on the r discount rate, and t the time period, leading to $(1+r)^t$. This is basiclly the discount formula of a simple zero-coupon bond. To get the present value, or price, of a bond paying multiple coupons plus the principal, the total present value is:

$$PV = \sum_{t=1}^{n} \frac{C}{(1+r)^t} + \frac{FV}{(1+r)^t} \tag{2.2}$$

where FV represents the face value of the bond, this is calculated over n periods.

2.2 Bond Price and Yield Relationship

There is a pattern that we all commonly know but most people never stop to think about the reason behind this behavior. When yields rise, existing bond prices fall, and vice versa.
This occurs mainly because:

- Bonds issued when rates are higher obviously offer better returns.

- Existing bonds with lower coupon rates become less attractive to investors.
- Prices must fall to make the existing bond's yield competitive with new issues.

Par, Discount, and Premium Bonds

Let's consider again the same bond as the one we performed computations on earlier. Since it was a 5% coupon rate and \$1000 face value bond, we can now distinguish between three scenarios:

- If the market yields are at 5%, the bond is said to trade **at par**. Its price will still be \$1000.
- If the market yields rise to 6%, the bond trades **at a discount**, with the associated price lower than \$1000.
- If the market yields fall to 4\$, the bond trades **at a premium**, so the price will be higher than \$1000.

This inverse relationship is not linear. It is *convex*, meaning that the price of a bond change more when yields fall than when they rise by the same amount.

2.3 Yield to Maturity (YTM)

The most important yield measure for bonds is the **Yield to Maturity** or **YTM**. This measure represents the total annualized return an investor earns if the purchase of the bond is made at its current price and is held until maturity, assuming all coupons are reinvested at the YTM rate, for example in similar bonds. More specifically, the YTM is the *Internal Rate of Retun (IRR)* that equates the present value of all future cash flows to the current bond price. It's calculated iteratively using the bond pricing formula:

$$P = \sum_{t=1}^{n} \frac{C}{(1 + \frac{YTM}{m})^t} + \frac{FV}{(\frac{YTM}{m})^n} \tag{2.3}$$

Where m is the number of coupon payments per year. More systematically and practically, traders use software (the most widespread is Bloomberg Terminal of course), that can compute Yield to Maturity instantly.
YTM can also be approximated by the simplified formula:

$$YTM \approx \frac{C + \frac{FV-P}{n}}{\frac{FV+P}{2}} \tag{2.4}$$

Obviously this provides a reasonable estimate that is less accurate than the iterative solution given above.

2.4 Other Yield Measures

While the Yield to Maturity (YTM) is the most commonly quoted measure of return, practitioners often consider alternative yield metrics that better capture specific features or risks of certain bonds. Each measure offers a slightly different lens on the same underlying idea: the relationship between price, coupon income, and time.

Current Yield

The **Current Yield (CY)** measures the bond's annual income relative to its current market price:

$$CY = \frac{\text{Annual Coupon Payment}}{P} \tag{2.5}$$

This ratio approximates the income return from holding the bond for one year, disregarding both the time value of money and any capital gain or loss at maturity.

For instance, if a bond with a \$50 annual coupon trades at \$950, its current yield is:

$$CY = \frac{50}{950} = 5.26\% \tag{2.6}$$

While CY is simple to compute, it ignores that the investor may also experience a price change as the bond approaches maturity (the price will generally converge toward par). Hence, it's mainly used for quick income comparisons between bonds, not as a comprehensive measure of total return.

Yield to Call (YTC)

Callable bonds allow the issuer to redeem (or "call") the bond before maturity, usually after a fixed lockout period and often at a specified **call price** (commonly above par).

The **Yield to Call (YTC)** is computed similarly to the YTM but assumes the bond will be called at the earliest possible date:

$$P = \sum_{t=1}^{n_c} \frac{C}{(1 - \frac{YTC}{2})^t} + \frac{F_{call}}{1 + \frac{YTC}{2}^{n_c}} \tag{2.7}$$

where n_c represents the number of semiannual periods until the first call date, and F_{call} the call price.

This measure is particularly relevant when interest rates fall, as issuers are more likely to refinance their debt at lower rates, cutting off the investor's future coupons. For this reason, YTC provides a more realistic estimate of return for callable securities in declining rate environments.

Yield to Worst (YTW)

The **Yield to Worst (YTW)** represents the most conservative expected return an investor could receive, assuming the issuer exercises all embedded options (such as calls, puts, or redemptions) in the investor's disfavor. Mathematically, it is the minimum of all possible yield outcomes:

$$YTW = min(YTM, YTC_1, YTC_2, \ldots) \tag{2.8}$$

For callable or putable bonds, the YTW highlights the "worst-case" scenario from a yield perspective.

In practice, professional traders and risk managers often quote YTW for corporate and structured products, as it accounts for both market risk and embedded optionality more conservatively than YTM or YTC alone.

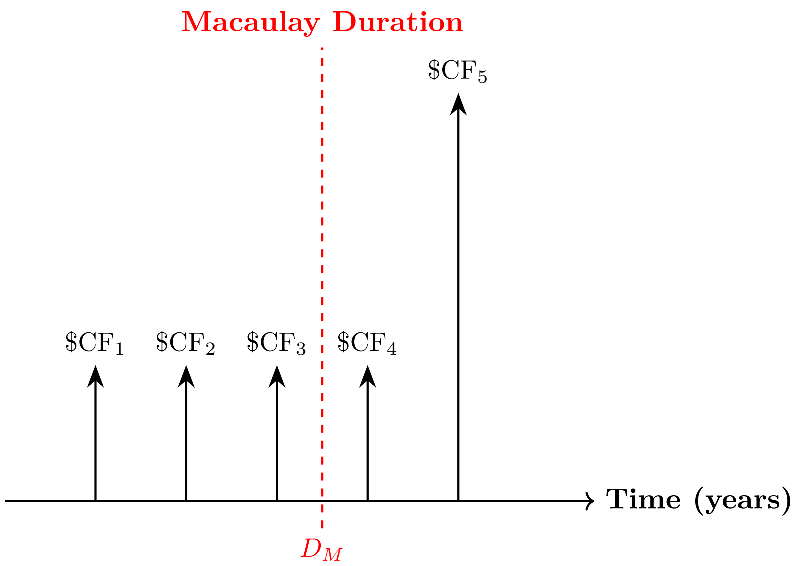
Chapter 3

Part III: Risk Metrics

3.1 Duration: Measuring Interest Rate Risk

Duration is the weighted average time until a bond's cash flows are received, and it serves as the primary measure of interest rate sensitivity. There are two ways to calculate the **duration** as a measure of interest rate risk.

- **Macaulay Duration** measures the weighted average time (in years) to receive the bond's cash flows:
$$D_{Mac} = \frac{\sum_{t=1}^{n} t \times \frac{C}{(1+y)^t} + n \times \frac{FV}{(1+y)^n}}{P} \tag{3.1}$$
Where t is time in years, y is the yield per period, and P is a bond price.



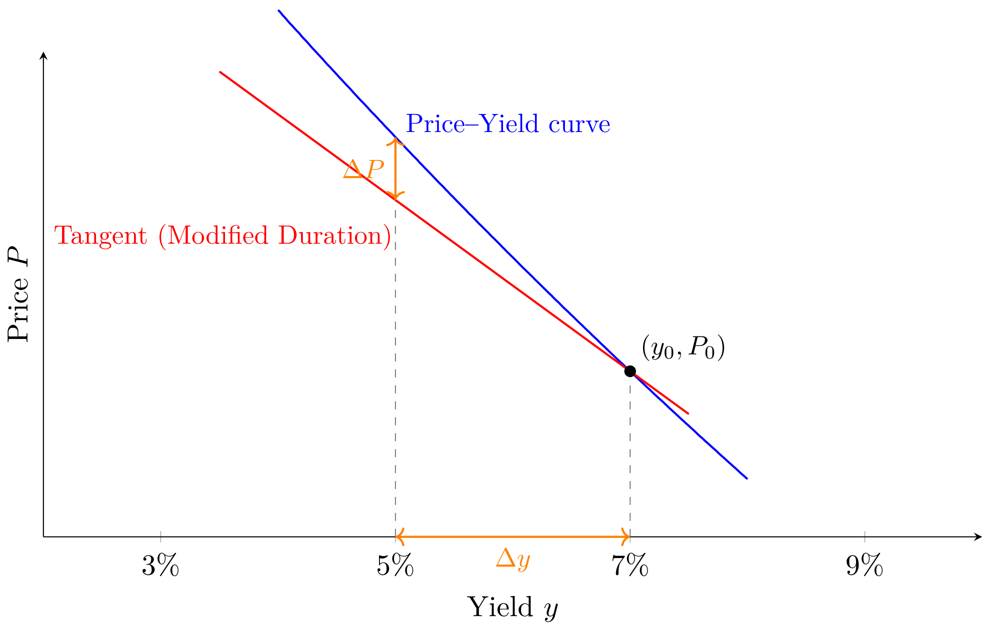
Macaulay Duration represents the weighted average time of the bond's discounted cash flows.
It is the "center of gravity" of its present value distribution.

FIGURE 3.1. Graphical intuition of Macaulay Duration.

- **Modified Duration** measures the percentage change in bond price for a 1% change in yield:

$$D_{Mod} = \frac{D_{Mod}}{1 + \frac{y}{m}} \tag{3.2}$$

Where m is the number of coupon payments per year.



An example application of **Modified Duration**:
If a bond has a modified duration of 4.5, a 1% increase/decrease in yields will cause the bond price to fall/rise by approximately 4.5%.

We can recap the key duration principles:

- Zero-coupon bonds have duration equal to their maturity.
- Higher coupon rates == lower duration, as more cash flows are received earlier in time.
- Longer maturity bonds have higher duration and greater interest rate risk.

3.2 Convexity: Beyond Linear Approximation

Duration assumes a linear relationship between price and yield changes, while empirically it is observed to be **convex**.
Convexity measures the curvature of the price-yield relationship and the *rate at which duration changes* as yields change.

Name	Measure	Actual	Limitation
Modified Duration	Linear price changes to small yield moves.	Estimate of 1% change in bond price keeping everything else constant.	Under/Over estimates price changes for large rate moves.
Convexity	Curvature in yield-price relationship.	Accounts for the convexity rather than linearity of the relationship.	Not really meaningful if not paired with duration.

TABLE 3.1. *Modified Duration* and *Convexity* differences.

$$\text{Duration} = \frac{1}{P} \times \frac{d^2 P}{dy^2} \tag{3.3}$$

We obviously can identify two regimes: **positive** and **negative** convexity.

Positive Convexity. When the convexity is positive[1], it means that bond prices increase more when yields fall than they decrease when yields rise by the same amount. This asymmetry benefits more the investors.

Negative Convexity. Usually happens in callable bonds and mortgage-backed securities, particularly when issuers can refinance. As yields fall, the issuer is more likely to call the bond or mortgage, effectively capping the upside price appreciation.
In practice, we can see the difference between **modified duration** and **convexity** more clearly in the Table 3.1.
For precise change estimates, both duration and convexity are used:

$$\Delta P \approx -D_{Mod} \times \Delta y + \frac{1}{2} \times \text{Convexity} \times (\Delta y)^2 \tag{3.4}$$

Where ΔP is the percentage price change and Δy is the yield change.

[1]Typical for option-free bonds.

Chapter 4

Part IV: Market Conventions and Price Denominations

4.1 Clean Price vs. Dirty Price

Bonds trade with two distinct pricing concepts that everyone (especially traders) must understand.

Clean Price. Also known as *Quoted Price* is the price of a bond **excluding** accrued interest. This is the actual price quoted in markets and used for comparison purposes. Clean prices provide stable reference points because they don't fluctuate daily due to the accrual of the interest.

Dirty Price. Also known as *Full Price* or *Invoice Price* is the total amount a buyer actually pays, including accrued interest. This is the effective **settlement** amount:

$$\text{Dirty Price} = \text{Clean Price} + \text{Accrued Interest} \tag{4.1}$$

Accrued interest is the interest that has accumulated since the last coupon payment date. When a bond is sold between coupon dates, the buyer needs to compensate the seller for the portion of the next coupon the seller has earned but *won't receive*. The specific calculation, which is a simple ratio between the coupon amount multiplied for the days since the last payment and the days into the coupon period, depends on the **day count convention** used.

4.2 Day Count Conventions

Day count conventions are standardized methods for calculating accrued interest and determining the fraction of a year between two dates, and different bond markets use different day count conventions. **Actual/Actual** convention uses actual number of days in both numerator and denominator. This is the most accurate method, and it is commonly used for U.S. Treasury bonds. It differs from the **30/360** day count convention because the latter assumes each month has 30 days and each year has 360 days, which simplifies calculation and it is used for U.S. corporate and municipal bonds. For example, January 15th to March 15th = 60 days, even if actual calendar days differ.

Another convention is **Actual/360** which uses the actual number of days but assumes 360-day years. Commonly used for money market instruments, T-Bills. On the other hand we also have **Actual/365** which intuitively uses the actual days with 365-day years, used in some markets for government bonds. This choice of convention affects accred interest

calculations and can lead to meaningful differences in settlement amounts, particularly for large institutional trades.

4.3 Settlement Conventions

The *settlement period* is the span of time occurring between the trade date and the actual transfer of ownership of the bond. Typically, bond trades settle on a **T+1** (Government Bonds) or **T+2** (Corporate Bonds) basis, meaning settlement occurs one or two business days after the trade date. During this period:

1. The trade is agreed upon at the **clean price**.
2. Accrued interest is calculated from the last coupon date to the **settlement date**.
3. The buyer pays the **dirty price** to the lender at settlement.

Chapter 5

Part V: Advanced Spread Measures

5.1 Credit Spreads

The **credit spread** is the additional yield investors demand over the risk-free rate (government bond yield) to compensate for credit risk (default or downgrade). It is calculated as the difference between the Treasury Yield and Corporate Bond Yield. For example, for a 10-year bond yielding 5% when the 10-year Treasury yields 3%, the credit spread is 200bp (2%).
Credit spreads vary based on:

- **Credit Rating**: lower-rated bonds have wider spreads.
- **Economic Conditions**: spreads widen during recessions as default risk increases.
- **Liquidity**: less liquid bonds command higher spreads.
- **Industry Sector**: some sectors are perceived as riskier.

5.2 Z-Spread (Zero-Volatility Spread)

The **Z-Spread**, or **Zero-Volatility Spread**, is the constant spread that, when added to each spot rate on the risk-free zero-coupon yield curve, makes the present value of a bond's cash flows equal to its market price. Unlike simple **credit spreads**, which use

a single benchmark yield, the Z-Spread discounts each cash flow at the appropriate spot rate plus a constant spread, making it more accurate for bonds with different features.

5.3 Options-Adjusted Spread (OAS)

For bonds with embedded options (callable bonds, putable bonds, mortgage-backed securities), the **Option-Adjusted Spread** or **OAS**provides the most accurate measure of value.

$$\text{OAS} = \text{ZSpread} - \Pi_{option} \tag{5.1}$$

We now take into consideration a *callable* and a *putable* bond. For the **callable** bond:

- The call option has value to the issuer (which must be negative for investors).
- The Option-Adjusted Spread is less than the Z-Spread.
- The difference represents the cost to investors of the call option.

On the other hand, for a bond that's **putable**, the put option has value to the investor, OAS $>$ Z-Spread. The actual computation of the OAS requires modeling interest rate scenarios (maybe Monte Carlo-based approaches) to account for the probability and timing of option exercise, while also keeping an eye on the volatility surface. In real-life scenarios, multiple models are embedded into systems like Bloomberg Terminal.

Chapter 6

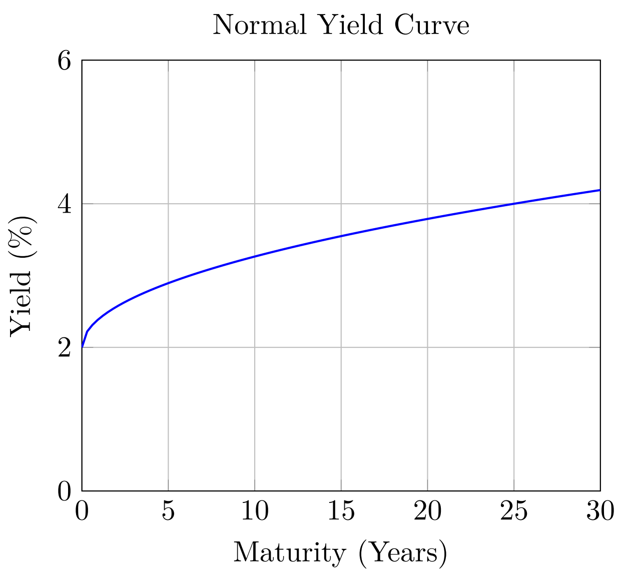
Part VI: Yield Curve and Bootstrapping

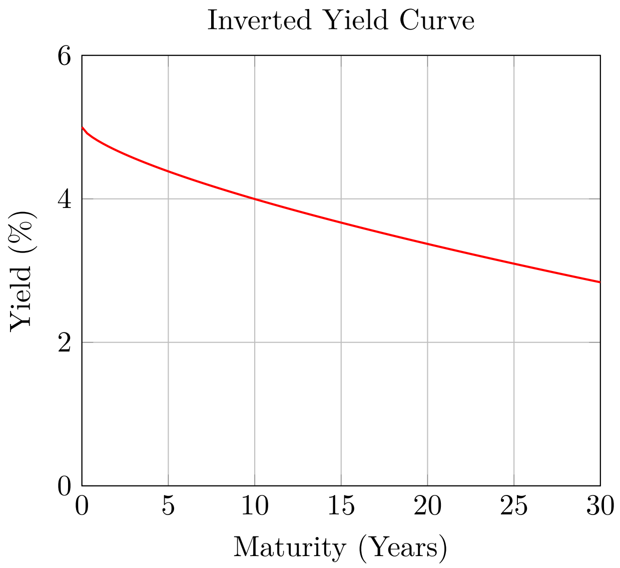
6.1 The Yield Curve

The **yield curve** plots yields of bonds with different maturities but similar credit quality, typically government bonds. The most direct insights about market expectations for interest rates can be extracted from the shape of the curve.

Normal (Upward-Sloping) Curve. When long-term yields exceed short-term yields, the curve reflects inflation expectation over time, along with a clear visualization of the risk premium assumed for longer maturity. This type of curve is expected and typical during *stable economic growth* periods.

Inverted (Downward-Sloping) Curve. Short-term yields exceed long term yields, signaling expectation of recession/economic slowdown, anticipating central bank rate cuts.





The **flat curve** highlights a period of economic uncertainty.

6.2 Bootstrapping the Zero-Coupon Curve

Bootstrapping is a method used to construct a zero-coupon yield curve from observable coupon-bearing bond prices. This zero curve is essential for accurate bond pricing and derivatives valuation, from a sequential process.

1. Get short-term zero-coupon rates (from T-Bills) to set initial spot rates.
2. Use coupon bonds with increasing maturities to solve for longer spot rates.
3. Apply forward substitution to extract each subsequent zero rate.

Let's assume a 6-month T-Bill, for which we pay \$9,785 now and receive \$10,000 at maturity.

$$\Pi^{Yield} = \frac{10000 - 9785}{9785} \times \frac{360}{180} = 0.0439448135 \approx 4.39\% \tag{6.1}$$

We then take into consideration longer maturities coupon bonds' rates[1].

- 2Y: 3.73% yield, 3.50% coupon.
- 5Y: 3.82% yield, 3.63% coupon.
- 10Y: 4.29% yield, 4.25% coupon.

Let's assume their price to be $P = 100$.
The first step is to compute the discount factor for the 6-month T-Bill:

$$D(0, 0.5) = \frac{9785}{10000} = 0.9785 \tag{6.2}$$

[1] Data fetched from Bloomberg.

The continuously compounded zero rate for 6 months is then:

$$r(0, 0.5) = -\frac{1}{0.5}\ln(0.9785) = 0.043468 \approx 4.35\% \tag{6.3}$$

To bootstrap, normally we would need an instrument maturing at 1Y, but since we didn't extract it, it is a nice example of how to handle these types of unknown values. We can interpolate a 1Y continuously compounded rate between our 6-month T-Bill and the 2Y bond. We know that $\Pi_2 = 3.73\%$, so:

$$r_2 \approx \ln(1 + y_2) = \ln(1 + 0.0373) = 0.03662118346 \approx 3.66\% \tag{6.4}$$

Now that we have the approximated continuously compounded rate, we can simply linearly interpolate between $t = 0.5$ and $t = 2$ to get the rate at 1Y:

$$r(0, 1) \approx r(0, 0.5) + \frac{1 - 0.5}{2 - 0.5}(r_2 - r(0, 0.5)) \tag{6.5}$$

$$r(0, 1) \approx 0.043468 + 0.333333(0.0366211 - 0.043468) = 0.04118570228 \approx 4.12\% \tag{6.6}$$

Then we can obtain the 1Y discount factor $D(0, 1)$:

$$D(0, 1) = e^{-r(0,1)\times 1} = e^{-0.04118570228} = 0.959650904 \approx 0.9596 \tag{6.7}$$

As we know, the coupon rate is $c_2 = 3.73\%$, while $P = 100$, hence the cashflow at maturity is equal to \$103.73. The pricing equation is then:

$$100 = 3.73 \times D(0, 1) + 103.73 \times D(0, 2) \tag{6.8}$$

and, since we know $D(0, 1)$ we can solve for $D(0, 2)$:

$$D(0, 2) = \frac{100 - 3.73 \times D(0, 1)}{103.73} \tag{6.9}$$

$$D(0, 2) = \frac{100 - 3.73(0.9596)}{103.73} = 0.929535255 \approx 0.9295 \tag{6.10}$$

then, $r(0, 2)$:

$$r(0, 2) = -\frac{1}{2}\ln(D(0, 2)) = 0.03655423591 \approx 3.65\% \tag{6.11}$$

So then, after recursively following this process, we end up with:

Maturity (Y)	$D(0,t)$ **Discount Factor**	$r(0,t)$ **Continuous Rate**
0.5	0.9785	4.35%
1	0.9596	4.12%
2	0.9295	3.65%
5	0.8290	3.75%
10	0.6570	4.20%

We can now see the curve derived from this computation:

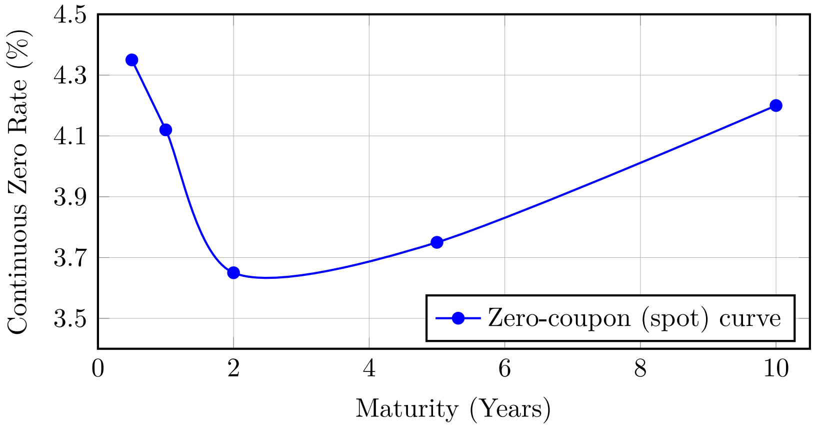


FIGURE 6.1. Zero-coupon yield curve bootstrapped from market data.

This bootstrapped zero curve is crucial in a number of areas. For example when pricing bonds with any maturity, valuing interest rate derivatives, calculating forward rates or determining arbitrage opportunities.

Chapter 7

Part VII: Repo Markets and Bond Financing

7.1 Repurchase Agreements (Repo)

A **Repurchase Agreement** or **Repo**, is a short-term *secured* loan where one party sells securities (typically government bonds) to another party with an agreement to repurchase them at an interest-adjusted price at a predetermined future date.
From the dealer's perspective, we can identify two types of repos:

- **Classic Repo**: the dealer borrows cash by temporarily selling securities.
- **Reverse Repo**: the dealer lends cash by temporarily buying securities.

3 important terms are to be known. We call the **collateral** the securities used to back the loan (in repos, usually Treasuries), the **repo rate** the implied interest to be applied on the cash loan, and an **haircut**, which is the difference betweem the market value of collateral and the loan amount, providing a cushion against price changes.

For example a dealer repos \$30 million par value of Treasury bonds with market value of \$31.2 million for 51 days, so:

- Municipality takes a 2% haircut, lending \$30.6 million at 5.25% repo rate.
- After 51 days, dealer returns the bonds and repays: $30.6 \times (1 + 0.0525 \times \frac{51}{360}) = 30.83$ milllion.

So, repos are used for mainly 4 purposes:

- **Financing**: dealers finance their bond inventories at favorable rates.
- **Short Selling**: traders borrow bonds via reverse repo to establish short positions.
- **Liquidity Management**: banks and institutional investors manage short-term cash needs.

- **Monetary Policy**: central banks use repo/reverse repo operations to control money supply.

This market is quite important in terms of size, some key figures at this time:

- Global outstanding: over $16T
- U.S. repo market size: ≈ $11.9T in 2024.
- European repo market size: ≈ €10.86T in 2024.
- Daily global turnover: about €3T.

Chapter 8

Part VIII: Credit Derivatives

8.1 Credit Default Swaps (CDS)

A **Credit Default Swap** (or **CDS**) is a derivative contract providing insurance against credit events such as bankruptcy, default, restructuring... of a reference entity.

The structure of a CDS is:

- **Protection Buyer**: pays periodic premiums to protection seller.
- **Protection Seller**: pays compensation of credit event occurs.
- **Notional Amount**: face value of debt being insured.
- **Premium (Spread)**: annual rate paid, typically 100 bps for investment-grade or 500 bps for high-yield.

For example, if an investor owns \$10 million of Company X's bonds and buys CDS protection:

1. Pays 150 bps (1.5%) annually, or \$150,000 per year.
2. If Company X defaults, receives $\$10,000,000 \times (1 - \text{Recovery Rate})$.
3. if RR is 40%, receives a \$6 million payout.

Pricing of a CDS. CDS' value equals the present value of expected payouts minus the present value of premium payments:

$$\text{CDS} = PV(\text{Protection Leg}) - PV(\text{Premium Leg}) \tag{8.1}$$

We now have that the *Protection Leg* depends on the **hazard rate** which is the PD given no prior default, the **recovery rate** (% of notional received after default), and the **discount factors** for PV calculation. CDSs are commonly used for *hedging, speculation, arbitrage*, or *capital relief*.

8.2 CDS Indexes

CDS Indexes are standardized pool of single-name CDS, providing liquid instruments for trading broad credit exposures.

CDX (North America). Involving CDX.IG which includes 125 investment-grade names, and CDX.HY, with 100 high-yield corporate names.

iTraxx (Europe/Asia). Involving iTraxx Europe (125 European investment-grade names), and iTraxx Crossover (75 sub-investment-grade European names). For Asian regions, iTraxx Japan, iTraxx Asian ex-Japan IG, and iTraxx Australia.
Indexes roll every six months (March and September), incorporating the most liquid credits. Index CDS volumes far exceed single-name CDS volumes, hence providing deep liquidity for hedging and relative value trading.

Chapter 9

Part IX: Practical Trading Applications

9.1 Trade Workflow on a Fixed Income Desk

This section is just something I found throguh shallow research, it doesn't claim to be accurate nor helpful.

1. **Pre-Trade Analysis**:
   - Monitor market data: yields, spreads, economic releases.
   - Review portfolio exposures and risk limits.
   - Identify trading opportunities based on proprietary strategy.
2. **Price Discovery**:
   - Liquid bonds: check prices.
   - Illiquid bonds: sen RFQs to multiple dealers.
   - Assess bid-ask spreads and available sizes.
3. **Risk Management**:
   - Calculate duration and convexity exposures.
   - Hedge interest rate risk with futures or swaps.
   - Monitor credit risk and concentration limits.
   - Adjust positions.

9.2 Common Trading Strategies

- **Carry Trade**: Buy higher-yielding bonds, finance via repo at lower rates, profit from the spread over time.
- **Curve Trade**: Take positions based on expected changes in yield curve shape:
  - *Steepener*: Long duration at one maturity, short at another, expecting curve to steepen.

  - *Flattener*: Opposite of steepener.

- **Credit Spread Trade**: Buy undervalued bonds with wide spreads, sell overvalued bonds with tight spreads, betting on spread convergence.
- **Basis Trade**: Exploit pricing differences between related instruments.
  - Cash bond vs. CDS on same issuer.
  - Bond vs. bond futures.
  - Physical bonds vs. ETFs.
- **Relative Value**: Identify similar bonds that are mispriced relative to each other, long the cheap bond and short the expensive bond.

Risk Management and P&L Attribution. Professional bond traders monitor several key metrics daily:

- **Duration Risk**: Interest rate sensitivity of the portfolio measured in years.
- **Convexity**: Non-linear interest rate risk, particularly important for large moves.
- **Credit Risk**: Exposure.
- **Spread Risk**: Sensitivity to changes in credit spreads.
- **Carry**: Expected P&L from holding positions assuming no price changes.
- **Mark-to-Market P&L**: Daily Gains/Losses.
- **DV01**: Change in portfolio value for a 1 bp yield move.

Chapter 10

X: Bonds Handbook

10.1 Bond Pricing & Yield

Clean Price (Quoted) : Excludes accrued interest.

Dirty Price (Invoice) : Clean Price + Accrued Interest.

Accrued Interest : $\frac{\text{Days Accrued}}{DaysinPeriod} \times \text{Coupon}$.

Yield to Maturity (YTM) : $P = \sum_{t=1}^{n} \frac{C}{(1+\frac{YTM}{m})^t} + \frac{FV}{(\frac{YTM}{m})^n}$.

Current Yield : $\frac{C}{P}$.

Spot Rate Extraction (Bootstrapping) : Extract zero-coupon rates from coupon bonds sequentially.

10.2 Duration & Convexity

Macaulay Duration : $D_{Mac} = \frac{\sum_{t=1}^{n} t \times \frac{C}{(1+y)^t} + n \times \frac{FV}{(1+y)^n}}{P}$.

Modified Duration : $D_{Mod} = \frac{D_{Mod}}{1+\frac{y}{m}}$.

Price Change Approximation : $\Delta P \approx -D_{Mod} \times \Delta y + \frac{1}{2} \times \text{Convexity} \times (\Delta y)^2$.

Dollar Duration (DV01) : $DV01 = D_{Mod} \times P \times 0.01\%$ loss per 1bp move.

Convexity : $\text{Convexity} = \frac{\sum t(t+1)PV(CF_t)}{P(1+y)^2}$.

Key Rate Duration : Sensitivity to specific yield curve tenors (2Y, 5Y...).

10.3 Spread Analysis

Nominal Spread : Bond YTM - Treasury YTM.

Zero-Volatility Spread (Z-Spread) : Parallel shift to spot curve that equals bond price.

Option-Adjusted Spread (OAS) : Z-Spread adjusted for embedded optionality.

I-Spread (Interpolated Spread) : vs. interpolated Treasury curve.

10.4 Yield Curve Positions

Bull Flattener : Buy long-dated, sell short-dated (duration long, curve bullish).

Bear Steepener : Sell long-dated, buy short-dated (duration short, curve bearish).

Bullet : Concentrate holdings at single maturity.

Barbell : Equal weights at short & long ends (convexity play).

10.5 Risk Metrics

PnL Attribution : $\text{PnL} = -DV01 \times \Delta y + \frac{1}{2} \times \text{Convexity} \times \text{Price} \times (\Delta y)^2 + \text{Carry} + \text{Curve Roll}$.

Curve Roll : Daily price appreciation toward maturity.

PVBP (Price Value of 1bp) : alternative to DV01.

Greeks :

- **Delta**: Duration exposure.
- **Gamma**: Convexity.
- **Theta**: Carry.
- **Vega**: Volatility exposure (for optionable bonds).

10.6 Arbitrage & Relative Value

Cash & Carry : Buy cash bond, short futures. Profit = $(\text{Bond Yield} - \text{Repo Rate}) \times Duration \times Time$.

Negative Carry Trade : When repo < yield (unusual, limited profit).

Curve Arbitrage : Exploit curve mispricings between maturities (or ”tenors”).

Quality Spread : Premium for high-grade vs. lower-grade bonds.

Butterfly Spread : Buy short & long, sell intermediate (a.k.a. *gamma play*) Profit = $(2 \times \text{Mid Price}) - (\text{Short Price} + \text{Long Price})$.

Implied Repo Rate (Cash & Carry) : $IRR = \frac{(P + \text{Accrued In}) \times (1+r) + C - (\text{Futures Price} \times \text{CF})}{\text{Bond Price}}$.

10.7 Hedging Strategies

Hedge Ratio : $\frac{DV01_{position}}{DV01_{hedge}}$.

Keyrate Hedging : Match individual tenor sensitivities independently.

Duration-Neutral Strategies : Equal long/short DV01.

Convexity Hedge : Use swaptions or options for large moves.

10.8 Repo & Financing

Repo Rate Formula : Seller finances purchase, rate drives carry profitability.

Haircut : % discount on collateral value for financing.

General Collateral (GC) Rate : Average repo rate.

Specific Collateral (SC) Rate : Reduced rate for on-the-run bonds (supply/demand).

Reverse Repo : Lend cash, borrow bonds.

10.9 Core Strategies

Riding the Yield Curve : Buy bond, hold to shorter maturity (simple price appreciation).

Steepener Trade : Long duration mismatch → Long 10Y vs. Short 2Y.

Flattener Trade : Short duration mismatch → Short 10Y vs. Long 2Y.

Par Swap : Exchange fixed coupon for floating rate.

Cross-Curve : Exploit correlation breaks between curve segments.

10.10 Market Convention Notes

U.S. Treasuries : Annual coupons, quotes in 32nd.

Corporates : Semi-annual coupons, quotes in decimals.

Gilts (UK) : Semi-annual coupons.

Bunds (DE) : Annual coupons.

Accrued Interest : Day count convention (Actual/Actual, 30/360...).

Settlement : T+2 (Treasuries), T+1 or T+2 (Corporates).